orginal-

$\sin\theta = \frac{\text{ভূম}}{\text{অতিভুজ}}$, $\cos\theta = \frac{\text{লম্ব}}{\text{অতিভুজ}}$, $\tan\theta = \frac{\text{লম্ব}}{\text{ভূমি}}$

$\sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta - \tan^2\theta = 1$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

90 এর গুনক বিজোড়

হয়: $\sin\theta \rightleftarrows \cos\theta$

$\tan\theta \rightleftarrows \cot\theta$

$\sec\theta \rightleftarrows \operatorname{cosec}\theta$

$\sin(-\theta) = -\sin\theta$

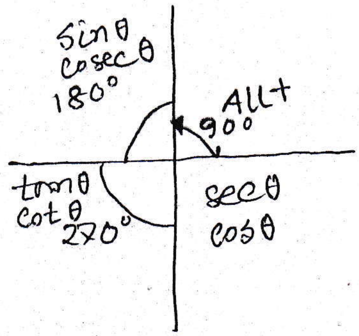
$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

$\tan(-\theta) = -\tan\theta$

$\cot(-\theta) = -\cot\theta$

$\sec(-\theta) = \sec\theta$

$\cos(-\theta) = \cos\theta$



8 (vi) সমাধান কর:-

$\sqrt{3}\cos\theta + \sin\theta = \sqrt{3}$

বা, $(\sqrt{3}\cos\theta) = \sqrt{3} - \sin\theta$

বা, $(\sqrt{3}\cos\theta)^2 = (\sqrt{3} - \sin\theta)^2$

বা, $(\sqrt{3})^2\cos^2\theta = (\sqrt{3})^2 - 2.\sqrt{3}\sin\theta + \sin^2\theta$

বা, $3\cos^2\theta = 3 - 2\sqrt{3}\sin\theta + \sin^2\theta$

বা, $3(1-\sin^2\theta) = 3 - 2\sqrt{3}\sin\theta + \sin^2\theta$

বা, $3 - 3\sin^2\theta = 3 - 2\sqrt{3}\sin\theta + \sin^2\theta$

বা, $3 - 3 = -2\sqrt{3}\sin\theta + \sin^2\theta + 3\sin^2\theta$

বা, $0 = -2\sqrt{3}\sin\theta + 4\sin^2\theta$

বা, $4\sin^2\theta - 2\sqrt{3}\sin\theta = 0$

বা, $2\sin\theta(2\sin\theta - \sqrt{3}) = 0$

$2\sin\theta = 0$ অথবা $2\sin\theta - \sqrt{3} = 0$

$\sin\theta = 0$ $\qquad$ $\sin\theta = \frac{\sqrt{3}}{2}$

$\sin\theta = \sin 0°$ $\qquad$ $\sin\theta = \sin 60°$

$\theta = 0$ $\qquad$ $\theta = 60°$

$\therefore \theta = 0, 60, 300, 360°$

মান নির্ণয় কর:-

5 (i) $\sin^2\frac{\pi}{7} + \sin^2\frac{5\pi}{14} + \sin^2\frac{8\pi}{7} + \sin^2\frac{9\pi}{14}$

$= \sin^2\frac{\pi}{7} + \sin^2\frac{5\pi}{14} + \sin^2(\frac{\pi}{2} + \frac{9\pi}{14}) + \sin^2\frac{9\pi}{14}$

$= \sin^2(\frac{\pi}{7}) + \sin^2(\frac{\pi}{2} \mp \frac{\pi}{7}) + \cos^2\frac{9\pi}{14} + \sin^2\frac{9\pi}{14}$

$= \sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7} + \cos^2\frac{9\pi}{14} + \sin^2\frac{9\pi}{14}$

$= 1 + 1$

$= 2$

$\frac{\pi}{2} + x = \frac{8\pi}{7}$

$x = \frac{8\pi}{7} - \frac{\pi}{2}$

$= \frac{16\pi - 7\pi}{14}$

$= \frac{9\pi}{14}$

$\frac{\pi}{2} + x = \frac{5\pi}{14}$

$x = \frac{5\pi}{14} - \frac{\pi}{2}$

$= \frac{5\pi - 7\pi}{14}$

$= \frac{-2\pi}{14}$

$x = -\frac{\pi}{7}$

৫. মান নির্ণয় কর:-

(iii) $\sin^2\frac{17\pi}{18} + \sin^2\frac{5\pi}{8} + \cos^2\frac{37\pi}{18} + \cos^2\frac{3\pi}{8}$

$= \sin^2\left(\pi - \frac{\pi}{18}\right) + \sin^2\left(\pi - \frac{3\pi}{8}\right) + \cos^2\left(2\pi + \frac{\pi}{18}\right) + \cos^2\frac{3\pi}{8}$

$= \sin^2\frac{\pi}{18} + \sin^2\frac{3\pi}{8} + \cos^2\frac{\pi}{18} + \cos^2\frac{3\pi}{8}$

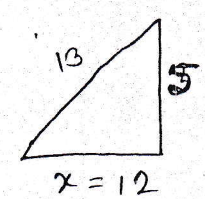
$= \sin^2\frac{\pi}{18} + \cos^2\frac{\pi}{18} + \sin^2\frac{3\pi}{8} + \cos^2\frac{3\pi}{8}$

$= 1 + 1$

$= 2$

৬. যদি $\sin\theta = \frac{5}{13}$ এবং $\frac{\pi}{2} < \theta < \pi$ হয়, তবে প্রমাণ কর যে,

$$\frac{\tan\theta + \sec(-\theta)}{\cot\theta + \operatorname{cosec}(-\theta)} = \frac{3}{10}$$



বামপক্ষ $\dfrac{\tan\theta + \sec\theta}{\cot\theta - \operatorname{cosec}\theta}$

$= \dfrac{-\frac{5}{12} - \frac{13}{12}}{-\frac{12}{5} - \frac{13}{5}}$

$= \dfrac{\frac{-5-13}{12}}{-\frac{12-13}{5}}$

$= \frac{-18}{12} \times \frac{5}{-25}$

$= \frac{3}{2} \times \frac{1}{5}$

$= \frac{3}{10}$

$\operatorname{cosec}\theta = \frac{13}{5}$

$\tan\theta = -\frac{5}{12}$

$\cot\theta = \frac{-12}{5}$

$\sec\theta = -\frac{13}{12}$

$5^2 + x^2 = 13^2$

$x^2 = 169 - 25$

$x^2 = 144$

$x = 12$

যৌগিক কোণের ত্রিকোণমিতিক অনুপাত অধ্যায় ১০

$\sin(A+B) = \sin A.\cos B + \cos A.\sin B$

$\sin(A-B) = \sin A.\cos B - \cos A.\sin B$

$\cos(A+B) = \cos A.\cos B - \sin A.\cos B$

$\cos(A-B) = \cos A.\cos B + \sin A.\sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A.\tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A.\tan B}$$

$$\cot(A+B) = \frac{\cot A.\cot B - 1}{\cot A + \cot B}$$

পৃষ্ঠা :- ২১৫ ৫. যদি $A+B = \frac{\pi}{4}$ তবে দেখাও যে

(i) $(1+\tan A)(1+\tan B) = 2$

(ii) $(\cot A - 1)(\cot B - 1) = 2$

দেওয়া আছে

$$A+B = \frac{\pi}{4}$$

$$\tan(A+B) = \tan\frac{\pi}{4}$$

$$\frac{\tan A + \tan B}{1 - \tan A.\tan B} = 1$$

$$\tan A + \tan B = 1 - \tan A.\tan B$$

L.H.S

$(1+\tan A)(1+\tan B)$

$= 1 + \tan A + \tan B + \tan A \tan B$

$= 1 + 1 - \tan A.\tan B + \tan A \tan B$

$= 2$

৮. যদি $A+B+C=\pi$ এবং $\cos A = \cos B . \cos C$ হয় তবে দেখাও যে $\tan A = \tan B + \tan C$.

R.H.S $\tan B + \tan C$	দেওয়া আছে
$= \dfrac{\sin B}{\cos B} + \dfrac{\sin C}{\cos C}$	$A+B+C=\pi$
	$\cos A = \cos B . \cos C$
$= \dfrac{\sin B . \cos C + \sin C . \cos B}{\cos B . \cos C}$	
	$B+C = \pi - A$
$= \dfrac{\sin (B+C)}{\cos A}$	
$= \dfrac{\sin(\pi - A)}{\cos A}$	
$= \dfrac{\sin A}{\cos A}$	
$= \tan A$	

৯. যদি $\sin\alpha \sin\beta - \cos\alpha\cdot\cos\beta + 1 = 0$ হয় তবে দেখাও যে $1 + \cot\alpha \tan\beta = 0$

দেওয়া আছে

$$\sin\alpha \sin\beta - \cos\alpha\cdot\cos\beta + 1 = 0$$

বা, $1 - \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta = 0$

বা, $1 - (\cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta) = 0$

বা, $1 - \cos(\alpha+\beta) = 0$

বা, $1 = \cos(\alpha+\beta)$

$\therefore \cos(\alpha+\beta) = 0$

$\therefore \sin^2(\alpha+\beta) + \cos^2(\alpha+\beta) = 1$

$\sin^2(\alpha+\beta) + 1^2 = 1$

$\sin^2(\alpha+\beta) = 1 - 1$

$\sin(\alpha+\beta) = 0$

$\sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta = 0$

$$\frac{\sin\alpha\cdot\cos\beta}{\sin\alpha\cdot\cos\beta} + \frac{\cos\alpha\cdot\sin\beta}{\sin\alpha\cdot\cos\beta} = 0$$

$\therefore 1 + \cot\alpha\cdot\tan\beta = 0$

রচনা:- ⑩ যদি $(\alpha+\beta) = \theta$ এবং $\tan\alpha = k\tan\beta$ হয় তবে দেখাও যে $\sin(\alpha-\beta) = \frac{k-1}{k+1}\sin\theta$

দেওয়া আছে

$\tan\alpha = k\tan\beta$

$\alpha+\beta = \theta$

$\therefore \tan\alpha = k\tan\beta$

$$\frac{\tan\alpha}{\tan\beta} = k$$

$$\Rightarrow \frac{\frac{\sin\alpha}{\cos\alpha}}{\frac{\sin\beta}{\cos\beta}} = k$$

$$\Rightarrow \frac{\sin\alpha}{\cos\alpha}\cdot\frac{\cos\beta}{\sin\beta} = k$$

5 ⑤

$$\frac{\sin\alpha.\cos\beta}{\cos\alpha.\sin\beta} = K$$

বা, $$\frac{\sin\alpha.\cos\beta - \cos\alpha.\sin\beta}{\sin\alpha.\cos\beta + \cos\alpha.\sin\beta} = \frac{K-1}{K+1}$$

বা, $$\frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)} = \frac{K-1}{K+1}$$

বা, $$\frac{\sin(\alpha-\beta)}{\sin\theta} = \frac{K-1}{K+1}$$

$$\sin(\alpha-\beta) = \frac{K-1}{K+1}\sin\theta$$

12. যদি $\tan\alpha + \tan\beta = b$, $\cot\alpha + \cot\beta = a$ এবং $\alpha+\beta = \theta$ হয় তবে দেখাও যে $(a-b)\tan\theta = ab$

$\therefore (a-b)\tan\theta = ab$

$$\tan\theta = \frac{ab}{a-b}$$

$\Rightarrow \tan\theta =$

$\Rightarrow \tan(\alpha+\beta)$

$$\Rightarrow \frac{\tan\alpha + \tan\beta}{1-\tan\alpha.\tan\beta}$$

$$= \frac{b}{1-\frac{b}{a}}$$

$$= \frac{b}{\frac{a-b}{a}}$$

$$= b \times \frac{a}{a-b}$$

$$\tan\theta = \frac{ab}{a-b}$$

দেওয়া আছে

$\alpha+\beta = \theta$

$\tan\alpha + \tan\beta = b$

$\cot\alpha + \cot\beta = a$

$$\cancel{\frac{1}{\cot\alpha}}\ \frac{1}{\tan\alpha} + \frac{1}{\tan\beta} = a$$

$$\Rightarrow \frac{\tan\beta + \tan\alpha}{\tan\alpha\tan\beta} = a.$$

$$\Rightarrow \frac{\tan\beta + \tan\alpha}{a} = \tan\alpha.\tan\beta$$

$$\frac{b}{a} = \tan\alpha.\tan\beta$$

$$\tan\alpha.\tan\beta = \frac{b}{a}$$

$(a-b)\tan\theta = ab$ (প্রমাণিত)

5

৮. যদি $A+B+C=\pi$ এবং $\cos A = \cos B.\cos C$ হয় তবে দেখাও যে $\tan A = \tan B + \tan C$.

R.H.S $\tan B + \tan C$

দেওয়া আছে

$A+B+C=\pi$

$\cos A = \cos B.\cos C$

$B+C = \pi - A$

$$= \frac{\sin B}{\cos B} + \frac{\sin C}{\cos C}$$

$$= \frac{\sin B.\cos C + \sin C.\cos B}{\cos B.\cos C}$$

$$= \frac{\sin(B+C)}{\cos A}$$

$$= \frac{\sin(\pi - A)}{\cos A}$$

$$= \frac{\sin A}{\cos A}$$

$$= \tan A$$

(7)

23. যদি $A+B+C=\pi$ এবং $\sin\left(A+\frac{C}{2}\right) = k\sin\frac{C}{2}$ হয় তবে প্রমাণ কর যে $\tan\frac{A}{2}\tan\frac{B}{2} = \frac{k-1}{k+1}$

দেওয়া আছে, $A+B+C=\pi$ $\Rightarrow$

$A+C = \pi - B$

$\frac{A+C}{2} = \frac{\pi-B}{2}$

$\frac{A+C}{2} = \frac{\pi}{2} - \frac{B}{2}$

$$\sin\left(A+\frac{C}{2}\right) = k\sin\frac{C}{2}$$

$$\frac{\sin\left(\frac{2A+C}{2}\right)}{\sin\frac{C}{2}} = k$$

$$\Rightarrow \frac{\sin\left(\frac{2A+C}{2}\right) - \sin\frac{C}{2}}{\sin\left(\frac{2A+C}{2}\right) + \sin\frac{C}{2}} = \frac{k-1}{k+1}$$

$$\Rightarrow \frac{2\cos\frac{\left(\frac{2A+C}{2}+\frac{C}{2}\right)}{2}.\sin\frac{\left(\frac{2A+C}{2}-\frac{C}{2}\right)}{2}}{2\sin\frac{\left(\frac{2A+C}{2}+\frac{C}{2}\right)}{2}.\cos\frac{\left(\frac{2A+C}{2}-\frac{C}{2}\right)}{2}} = \frac{k-1}{k+1}$$

$$\Rightarrow \frac{\cos\frac{\left(\frac{2A+C+C}{2}\right)}{2}.\sin\left(\frac{\frac{2A+C-C}{2}}{2}\right)}{2\sin\left(\frac{\frac{2A+C+C}{2}}{2}\right.\cos\frac{\frac{2A+C-C}{2}}{2}} = \frac{k-1}{k+1}$$

$$\Rightarrow \frac{\cos\frac{\frac{2A+2C}{2}}{2}\sin\frac{\left(\frac{2A}{2}\right)}{2}}{\sin\frac{\frac{2A+2C}{2}}{2}\cos\frac{\frac{2A}{2}}{2}} = \frac{k-1}{k+1}$$

$$= \frac{\cos\left(\frac{A+C}{2}\right)\sin\frac{A}{2}}{\sin\frac{A+C}{2}\cos\frac{A}{2}} = \frac{k-1}{k+1}$$

$$= \frac{\cos\left(\frac{\pi}{2}-\frac{B}{2}\right)\sin\frac{A}{2}}{\sin\left(\frac{\pi}{2}-\frac{B}{2}\right)\cos\frac{A}{2}} = \frac{k-1}{k+1}$$

$$\Rightarrow \frac{\sin\frac{B}{2}\sin\frac{A}{2}}{\cos\frac{B}{2}\cos\frac{A}{2}} = \frac{k-1}{k+1}$$

$$\tan\frac{B}{2}.\tan\frac{A}{2} = \frac{k-1}{k+1}$$

6

অধ্যায় - ১২

প্রয়োজনীয় সূত্র :-

$\sin(A+B) + \sin(A-B) = 2\sin A.\cos B$

$\sin(A+B) - \sin(A-B) = 2\cos A.\sin B$

$\cos(A+B) + \cos(A-B) = 2\cos A.\cos B$

$\cos(A-B) - \cos(A+B) = 2\sin A.\sin B$

$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

$\cos C - \cos D = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$

পৃষ্ঠা - 227

~~সহজ~~ রচনামূলক :- $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$

$= \frac{\sqrt{3}}{2}\sin 20 \sin 40 \sin 80$

$= \frac{\sqrt{3}}{2}\frac{1}{2}.2\sin 20^\circ \sin 40^\circ \sin 80^\circ$

$= \frac{\sqrt{3}}{2}\frac{1}{2}.\sin 20^\circ \quad 2\sin 40^\circ \sin 80^\circ$

$= \frac{\sqrt{3}}{2}.\frac{1}{2}\sin 20 \{\cos(40-80) - \cos(40+80)\}$

$= \frac{\sqrt{3}}{4}\sin 20 \{\cos(-40) - \cos 120^\circ\}$

$= \frac{\sqrt{3}}{4}\sin 20^\circ \cos 40 - \frac{\sqrt{3}}{4}\sin 20.\cos 120^\circ$

$= \frac{\sqrt{3}}{4}\sin 20 \cos 40 - \frac{\sqrt{3}}{4}\sin 20\left(\frac{-1}{2}\right)$

$= \frac{\sqrt{3}}{4}\sin 20 \cos 40 + \frac{\sqrt{3}}{8}\sin 20$

$= \frac{\sqrt{3}}{4}\sin 20 \cos 40 + \frac{\sqrt{3}}{4}\sin 20^\circ$

$= \frac{\sqrt{3}}{4}.\frac{1}{2}.2\sin 20 \cos 40 + \frac{\sqrt{3}}{4}\sin 20^\circ$

$= \frac{\sqrt{3}}{8}\{\sin(20+40) + \sin(20-40)\} + \frac{\sqrt{3}}{4}\sin 20^\circ$

$= \frac{\sqrt{3}}{8}\sin 60 + \frac{\sqrt{3}}{8}\sin(-20) + \frac{\sqrt{3}}{4}\sin 20$

$= \frac{\sqrt{3}}{8}.\frac{\sqrt{3}}{2} - \cancel{\frac{\sqrt{3}}{8}\sin 20} + \cancel{\frac{\sqrt{3}}{4}}\sin 20$

$= \frac{3}{\ldots}$

অধ্যায়-১২

$\sin 2A = 2\sin A \cdot \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$1 + \cos 2A = 2\cos^2 A$

$1 - \cos 2A = 2\sin^2 A$

$\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

পৃষ্ঠা ২৪: 241

সংক্ষিপ্ত :- 9. প্রমাণ কর যে $\tan 2A = (\sec 2A + 1)\sqrt{\sec^2 A - 1}$

R.H.S $(\sec 2A + 1)\sqrt{\sec^2 A - 1}$

$= \left(\dfrac{1}{\cos 2A} + 1\right)\sqrt{\tan^2 A}$

$= \left(\dfrac{1+\cos 2A}{\cos 2A}\right)\tan A$

$= \dfrac{2\cos^2 A}{\cos 2A} \cdot \dfrac{\sin A}{\cos A}$

$= \dfrac{2\cos A \cdot \sin A}{\cos 2A}$

$= \dfrac{\sin 2A}{\cos 2A}$

$= \tan 2A$

$\therefore$ $\cancel{\sec^2 A + 1}$

$\sec^2 A - \tan^2 A = 1$

$\sec^2 A - 1 = \tan^2 A$

15. দেখাও যে $\sec\theta = \dfrac{2}{\sqrt{2+\sqrt{2+2\cos 4\theta}}}$

L.H.S $= \dfrac{2}{\sqrt{2+\sqrt{2(1+\cos 4\theta)}}}$

$= \dfrac{2}{\sqrt{2+\sqrt{2 \cdot 2\cos^2 2\theta}}}$

$= \dfrac{2}{\sqrt{2+2\cos 2\theta}}$

$= \dfrac{2}{\sqrt{2(1+\cos 2\theta)}}$

$= \dfrac{2}{\sqrt{2 \cdot 2\cos^2\theta}}$

$= \dfrac{2}{\sqrt{4\cos^2\theta}}$

$= \dfrac{2}{2\cos\theta}$

$= \dfrac{1}{\cos\theta} = \sec\theta$

৭

15. ~~প্র~~. যদি $\tan\theta = \frac{1}{2}$ এবং $\tan\varphi = \frac{1}{3}$ তবে দেখাও যে, $\sin 2\theta = \cos 2\varphi$

L.H.S $\quad \sin 2\theta$

$$= \frac{2\tan\theta}{1+\tan^2\theta}$$

$$= \frac{2\cdot\frac{1}{2}}{1+\left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{1+\frac{1}{4}}$$

$$= \frac{1}{\frac{4+1}{4}}$$

$$= \frac{4}{5}$$

R.H.S $\quad \cos 2\varphi$

$$= \frac{1-\tan^2\varphi}{1+\tan^2\varphi}$$

$$= \frac{1-\left(\frac{1}{3}\right)^2}{1+\left(\frac{1}{3}\right)^2}$$

$$= \frac{1-\frac{1}{9}}{1+\frac{1}{9}}$$

$$= \frac{\frac{9-1}{9}}{\frac{9+1}{9}}$$

$$= \frac{\cancel{8}^{4}}{\cancel{9}} \times \frac{\cancel{9}}{\cancel{10}_{5}}$$

$$= \frac{4}{5}$$

$\therefore$ L.H.S = R.H.S

16. যদি $\tan\theta = \frac{1}{2}$ হয় তবে দেখাও যে $10\sin 2\theta - 6\tan 2\theta + 5\cos 2\theta = 3$

R.H.S $\quad 10.\sin 2\theta - 6\tan 2\theta + 5\cos 2\theta$

$$= 10.\frac{2\tan\theta}{1+\tan^2\theta} - 6.\frac{2\tan\theta}{1-\tan^2\theta} + 5.\frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$= 10.\frac{2.\frac{1}{2}}{1+\left(\frac{1}{2}\right)^2} - 6\,\frac{2.\frac{1}{2}}{1 - \left(\frac{1}{2}\right)^2} + 5\,\frac{1-\left(\frac{1}{2}\right)^2}{1+\left(\frac{1}{2}\right)^2}$$

$$= 10\cdot\frac{1}{1+\frac{1}{4}} - 6.\frac{1}{1-\frac{1}{4}} + 5.\frac{1-\frac{1}{4}}{1+\frac{1}{4}}$$

$$\cancel{=10.} = \frac{10}{\frac{4+1}{4}} - \frac{6}{\frac{4-1}{4}} + .5\,\frac{\frac{4-1}{4}}{\frac{4+1}{4}}$$

$$= \frac{\cancel{40}^{8}}{\cancel{5}} - \frac{\cancel{24}^{8}}{\cancel{3}} + \cancel{5}.\frac{3}{\cancel{4}}\times\frac{\cancel{4}}{\cancel{5}}$$

$$= 8 - 8 + 3$$

$$= 3$$

২২ নং 19

<u>রচনামূলক</u> :- যদি $\tan\alpha = \frac{1}{7}$ এবং $\tan\beta = \frac{1}{3}$ হয় তবে দেখাও যে $\cos 2\alpha = \sin 4\beta$

$\cos 2\alpha = \sin 4\beta$

বামপক্ষ. $\cos 2\alpha$

$$= \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$$

$$= \frac{1-\left(\frac{1}{7}\right)^2}{1+\left(\frac{1}{7}\right)^2}$$

$$= \frac{1-\frac{1}{49}}{1+\frac{1}{49}}$$

$$= \frac{\frac{49-1}{49}}{\frac{49+1}{49}}$$

$$= \frac{48}{\cancel{49}} \times \frac{\cancel{49}}{50}$$

$$= \frac{48}{50}$$

$$= \frac{24}{25}$$

ডানপক্ষ $\sin 4\beta$

$$= \sin 2.2\beta$$

$$= 2\sin 2\beta . \cos 2\beta$$

$$= 2 . \frac{2\tan\beta}{1+\tan^2\beta} \times \frac{1-\tan^2\beta}{1+\tan^2\beta}$$

$$= 2 . \frac{2 . \frac{1}{3}}{1+\left(\frac{1}{3}\right)^2} \times \frac{1-\left(\frac{1}{3}\right)^2}{1+\left(\frac{1}{3}\right)^2}$$

$$= 2 . \frac{\frac{2}{3}}{\frac{9+1}{9}} \times \frac{\frac{9-1}{9}}{\frac{9+1}{9}}$$

$$= 2 . \frac{2}{\cancel{3}} \times \frac{\overset{3}{\cancel{9}}}{10} \times \frac{8}{\cancel{9}} \times \frac{\cancel{9}}{10}$$

$$= \frac{\overset{2}{\cancel{4}}}{\underset{5}{\cancel{10}}} \times \frac{8.3}{\underset{5}{\cancel{10}}}$$

$$= \frac{24}{25}$$

22. যদি $\tan^2\theta = 1 + \tan^2\varphi$ হয়, তবে প্রমাণ কর যে $\cos 2\varphi = 1 + 2\cos 2\theta$

দেওয়া আছে,

$$\tan^2\theta = 1 + 2\tan^2\varphi$$

$$2\tan^2\varphi = \tan^2\theta - 1$$

$$\tan^2\varphi = \frac{\tan^2\theta - 1}{2}$$

L.H.S $\cos 2\varphi = \frac{1-\tan^2\varphi}{1+\tan^2\varphi}$

$$= \frac{1-\left(\frac{\tan^2\theta-1}{2}\right)}{1+\frac{\tan^2\theta-1}{2}}$$

$$= \frac{\frac{2-\tan^2\theta+1}{2}}{\frac{2+\tan^2\theta-1}{2}}$$

$$= \frac{3-\tan^2\theta}{1+\tan^2\theta}$$

$$= \frac{1+\tan^2\theta+2-2\tan^2\theta}{1+\tan^2\theta}$$

$$= \frac{1+\tan^2\theta}{1+\tan^2\theta} + \frac{2(1-\tan^2\theta)}{1+\tan^2\theta}$$

$$= 1 + 2\cos\theta$$

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(11)

২৩. যদি $2\tan\alpha = 3\tan\beta$ হয় তবে প্রমাণ কর যে $\tan(\alpha-\beta) = \dfrac{\sin 2\beta}{5-\cos 2\beta}$

দেওয়া আছে, $2\tan\alpha = 3\tan\beta$

$$\tan\alpha = \frac{3\tan\beta}{2}$$

$$\therefore \text{L.H.S } \tan(\alpha+\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha.\tan\beta}$$

$$= \frac{\frac{3\tan\beta}{2} - \tan\beta}{1+\frac{3\tan\beta}{2}.\tan\beta}$$

$$= \frac{\frac{3\tan\beta - 2\tan\beta}{2}}{\frac{2+3\tan^2\beta}{2}}$$

$$= \frac{\tan\beta}{\cancel{2}} \times \frac{\cancel{2}}{2+3\tan^2\beta}$$

$$= \frac{\frac{\sin\beta}{\cos\beta}}{2+\frac{3\sin^2\beta}{\cos^2\beta}}$$

$$= \frac{\frac{\sin\beta}{\cos\beta}}{\frac{2\cos^2+3\sin^2\beta}{\cos^2\beta}}$$

$$= \frac{\sin\beta}{\cancel{\cos\beta}} \times \frac{\cancel{\cos^2\beta}\cos\beta}{2\cos^2\beta+3\sin^2\beta}$$

$$= \frac{2\sin\beta.\cos\beta}{2(2\cos^2\beta+3\sin^2\beta)}$$

$$= \frac{\sin 2\beta}{4\cos^2\beta+6\sin^2\beta}$$

$$= \frac{\sin 2\beta}{5(\sin^2\beta+\cos^2\beta)-(\cos^2\beta-\sin^2\beta)}$$

$$= \frac{\sin 2\beta}{5-\cos 2\beta}$$

24. যদি α ও β কোণদ্বয় ধনাত্মক ও সূক্ষ্মকোণ হয় এবং $\cos 2\alpha = \frac{3\cos 2\beta - 1}{3 - \cos 2\beta}$ হয় তবে প্রমাণ কর $\tan\alpha = \sqrt{2}\tan\beta$

দেওয়া আছে,

$$\cos 2\alpha = \frac{3\cos 2\beta - 1}{3 - \cos 2\beta}$$

$$\frac{1-\tan^2\alpha}{1+\tan^2\alpha} = \frac{3 \cdot \frac{1-\tan^2\beta}{1+\tan^2\beta} - 1}{3 - \frac{1-\tan^2\beta}{1+\tan^2\beta}}$$

$$= \frac{\frac{3 - 3\tan^2\beta - 1 - \tan^2\beta}{1+\tan^2\beta}}{\frac{3 + 3\tan^2\beta - 1 + \tan^2\beta}{1+\tan^2\beta}}$$

$$= \frac{2 - 4\tan^2\beta}{\cancel{1+\tan^2\beta}} \times \frac{\cancel{1+\tan^2\beta}}{2 + 4\tan^2\beta}$$

$$\frac{1-\tan^2\alpha}{1+\tan^2\alpha} = \frac{2 - 4\tan^2\beta}{2 + 4\tan^2\beta}$$

$$\frac{1-\cancel{\tan^2\alpha}+1+\cancel{\tan^2\alpha}}{\cancel{1}-\tan^2\alpha-\cancel{1}-\tan^2\alpha} = \frac{\cancel{2}-\cancel{4\tan^2\beta}+\overset{2}{\cancel{2}}+\cancel{4\tan^2\beta}}{2-4\tan^2\beta-2-4\tan^2\beta}$$

$$\frac{\cancel{2}}{-\cancel{2}\tan^2\alpha} = \frac{\cancel{2\tan^2\beta}\ \cancel{4}}{-\underset{2}{\cancel{8}}\tan^2\beta}$$

$$\frac{1}{\tan^2\alpha} = \frac{1}{2\tan^2\beta}$$

$$\tan^2\alpha = 2\tan^2\beta$$

$$\tan\alpha = \pm\sqrt{2}\tan\beta$$

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25. যদি $\cos A + \cos B + \cos C = 0$ হয় তবে প্রমাণ কর যে $\cos 3A + \cos 3B + \cos 3C = 12 \cos A \cdot \cos B \cdot \cos C$

দেওয়া আছে. $\cos A + \cos B + \cos C = 0$

বামপক্ষ : $\cos 3A + \cos 3B + \cos 3C$

$= 4\cos^3 A - 3\cos A + 4\cos^3 B - 3\cos B + 4\cos^3 C - 3\cos C$

$= 4(\cos^3 A + \cos^3 B + \cos^3 C) - 3(\cos A + \cos B + \cos C)$

$= 4(\cos^3 A + \cos^3 B + \cos^3 C) - 3\times 0$

$= 4(\cos^3 A + \cos^3 B + \cos^3 C) - 12\cos A.\cos B.\cos C + 12\cos A.\cos B.\cos C$

$= 4\{\cos^3 A + \cos^3 B + \cos^3 C - 3\cos A.\cos B.\cos C\} + 12\cos A.\cos B.\cos C$

$= 4\{(\cos A + \cos B + \cos C)(\cos^2 A + \cos^2 B + \cos^2 C - \cos A.\cos B - \cos B.\cos C - \cos C.\cos A)\}$
$+ 12\cos A.\cos B.\cos C$

$= 4\times 0 + 12\cos A.\cos B.\cos C$

$= 12\cos A.\cos B.\cos C$

অধ্যায়-১২

(i) $\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$

(ii) $\tan\frac{C-A}{2} = \frac{c-a}{c+a}\cot\frac{B}{2}$

(iii) $\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$

আমরা জানি,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

আমরা জানি,

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c\sin B = b\sin C$$

$$\frac{\sin B}{\sin C} = \frac{b}{c}$$

$$\Rightarrow \frac{\sin B - \sin C}{\sin B + \sin C} = \frac{b-c}{b+c}$$

$$\Rightarrow \frac{2\cos\frac{B+C}{2}\cdot\sin\frac{B-C}{2}}{2\sin\frac{B+C}{2}\cdot\cos\frac{B-C}{2}} = \frac{b-c}{b+c}$$

$$\Rightarrow \cot\frac{B+C}{2}\cdot\tan\frac{B-C}{2} = \frac{b-c}{b+c}$$

$$\Rightarrow \frac{1}{\tan\frac{B+C}{2}}\cdot\tan\frac{B-C}{2} = \frac{b-c}{b+c}$$

$$\Rightarrow \tan\frac{B-C}{2} = \frac{b-c}{b+c}\tan\frac{B+C}{2}$$

(iv) ABC ত্রিভুজে $b=\sqrt{3}$ $c=1$ এবং $A=30$ হলে ত্রিভুজটি সমাধান কর

দেওয়া আছে

$b = \sqrt{3}$

$c = 1$

$A = 30$

আমরা জানি,

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos 30 = \frac{(\sqrt{3})^2+1^2-a^2}{2\cdot\sqrt{3}\cdot 1}$$

$$\frac{\sqrt{3}}{2} = \frac{3+1-a^2}{2\sqrt{3}}$$

$$2(4-a^2) = 2\cdot\sqrt{3}\cdot\sqrt{3}$$

$$4-a^2 = 3$$

$$4-3 = a^2$$

$$a^2 = 1$$

$$a = 1$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{1}{\sin 30} = \frac{1}{\sin C}$$

$$\sin C = \sin 30$$

$$C = 3$$

$$A+B+C = \pi$$

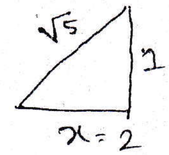
$$30+B+30 = \pi$$

$$B = 180-60$$

$$B = 120^\circ$$

অধ্যায় - ২১

প্রমাণ কর যে $4(\cot^{-1}3 + \operatorname{cosec}^{-1}\sqrt{5}) = \pi$



$$\cot^{-1}3 + \operatorname{cosec}^{-1}\sqrt{5} = \frac{\pi}{4}$$

$$= \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$$

$$= \tan^{-1}\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\cdot\frac{1}{2}}$$

$$= \tan^{-1}\frac{\frac{2+3}{6}}{\frac{6-1}{6}}$$

$$= \tan^{-1}\frac{5}{6}\times\frac{6}{5}$$

$$= \tan^{-1}1$$

$$= \frac{\pi}{4}$$

$x^2 + 1^2 = (\sqrt{5})^2$

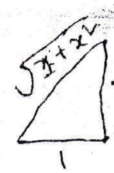
$x^2 = 5 - 1$

$x^2 = 4$

$x = 2$

* প্রমাণ কর যে $\sin\cot^{-1}\cos\tan^{-1}x = \sqrt{\frac{1+x^2}{2+x^2}}$

$$= \sin\cot^{-1}\cos\cos^{-1}\frac{1}{\sqrt{1+x^2}}$$



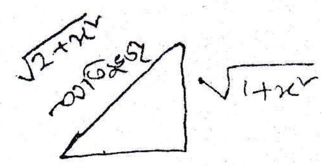
$$= \sin\cot^{-1}\frac{1}{\sqrt{1+x^2}}$$

(অতিভুজ)$^2 = 1^2 + x^2$

$$= \sin\sin^{-1}\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$$

অতিভুজ $= \sqrt{1^2+x^2}$

$$= \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$$



(অতিভুজ)$^2 = 1^2 + (\sqrt{1+x^2})^2$

$= 1 + 1 + x^2$

অতিভুজ $= \sqrt{2+x^2}$

৮. যদি $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$ হয় তবে প্রমাণ কর যে $xy + yz + zx = 1$

আমরা জানি,

$$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$$

$$\tan^{-1}\frac{x+y+z}{1-xy-yz-zx} = \frac{\pi}{2}$$

$$\frac{x+y+z}{1-xy-yz-zx} = \tan\frac{\pi}{2}$$

$1 - xy - yz - zx = 0$

$xy + yz + zx$